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QUANTUM PHASE UNCERTAINTIES IN THE CLASSICAL LIMIT

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Abstract

Several sources of phase noise, including spontaneous emission noise and the loss of coherence due to which-path information, are examined in the classical limit of high field intensities. Although the origin of these effects may appear to be quantum-mechanical in nature, it is found that classical analogies for these effects exist in the form of chaos.

1. Introduction

There are several sources of phase noise that may appear to be inherently quantum-mechanical in nature. One example is spontaneous emission noise, which is often attributed to vacuum fluctuations. Another example is the loss of coherence in which-path experiments, which can be shown to be due to the entanglement of one particle with another.

This paper addresses the question of whether or not these effects continue to exist in the macroscopic limit of high-intensity fields. If so, do they agree with the predictions of classical physics in that limit?

One motivation for considering these questions is to gain further insight into the origin of these effects. In addition, any disagreement with classical physics in the macroscopic limit would suggest an interesting experimental test of quantum mechanics in a new and untested situation.

It will be found that a classical analysis of these systems does give analogous effects due to classical chaos. This suggests that there is at least a loose connection between quantum noise and classical chaos.

On the other hand, classical physics cannot provide any analogy for nonlocal effects such as violations of Bell's inequality. The generalization of two-photon interferometry to high-intensity fields will be briefly discussed as an example of a situation in which no classical description exists even in the macroscopic limit.

2. Which-Path Experiments

Wave-particle duality suggests that we cannot determine the path that a particle has taken through an interferometer without destroying the interference pattern. In most cases, it can be shown that the loss of coherence is actually due to the entanglement of the particle's wave function with a second particle or system located in one path or the other. No actual observation of the path taken is necessary in order to eliminate the interference pattern. An interesting feature of these which-path experiments is that it is often possible to restore the interference pattern using a "quantum eraser"[1].

An excellent example of a which-path experiment is shown in Figure 1. As suggested by Scully[2] et al., a single atom is incident upon a beam splitter that divides its wave function along two separated paths. A microwave cavity is located in each path and is coupled to the atom in such a way that a low-energy microwave photon will be emitted into whichever cavity the atom passes through.

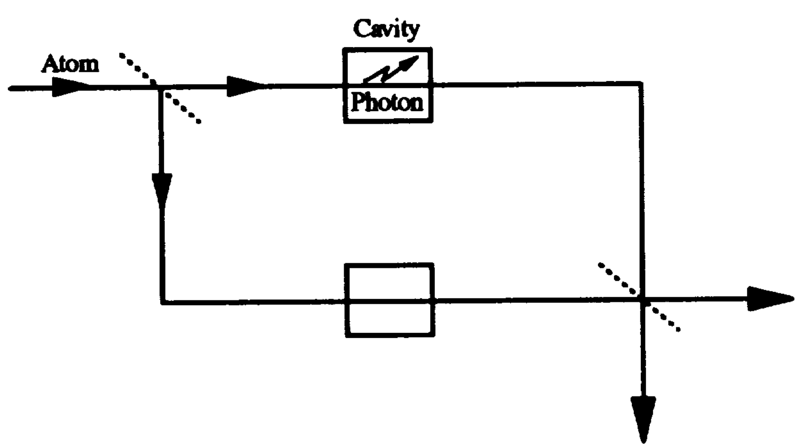

Fig. 1. A which-path experiment suggested by Scully et al. (Ref. 2) in which a microwave cavity is located in each arm of an atomic interferometer.

The interference pattern must be destroyed, since the path of the atom can be determined by detecting the location of the photon.

It can be shown that the change in the center-of-mass wave function of the atom has no significant effect and that the coherence is destroyed by the entanglement of the atom with the photon.

It is obvious that there can be no classical analogy for this kind of which-path experiment because an atom cannot be described by a wave in classical physics. But this begs the question of what is really responsible for the loss of coherence.

In order to allow a comparison with classical physics, consider instead the situation shown in Figure 2 in which the roles of the atom and photon have been interchanged[3]. Now a single photon is incident upon a beam splitter and its wave function propagates along two separated paths. A thin chamber containing gas atoms is located in each path and it is assumed that the photon is inelastically scattered, producing a secondary photon of low energy. The initial photon propagates with somewhat reduced energy toward a beam splitter and a single-photon detector. Once again, it can be shown that the change in the photon's wave function is irrelevant as long as $\delta k \delta x \ll \pi$, where δk is the change in wave number and δx is the thickness of the two chambers. The advantage of this which-path experiment is that it does allow a classical analysis if a large number of photons are incident, which corresponds to a classical light wave.

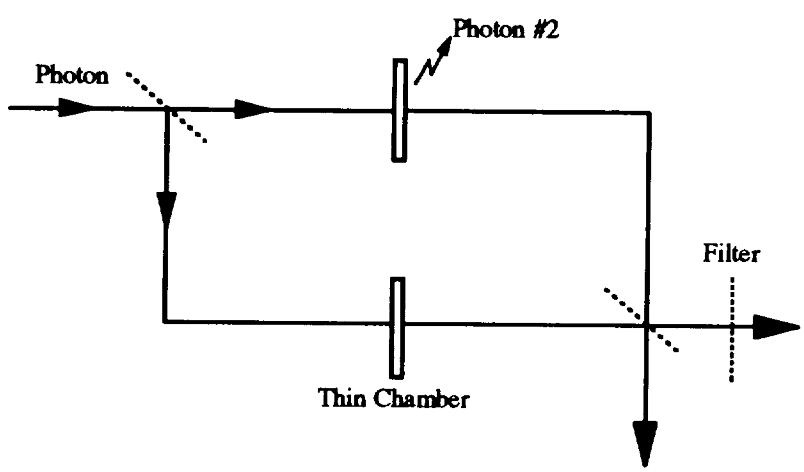

Fig. 2. A modified which-path experiment in which the roles of the atoms and photons have been interchanged to allow a comparison with classical physics.

The quantum-mechanical calculation is straightforward and has been described in more detail elsewhere[3]. Consider an operator $p^\dagger$

that creates a single photon in a short gaussian wave packet:

$$p^\dagger = \sum_i \beta_i a_i^\dagger \tag{1}$$

Here β_i are complex coefficients and $a_i^\dagger$ creates a photon of frequency ω_i. In order to achieve the macroscopic limit of high intensities, the quantum state will be taken to be a coherent state of the form

$$|\Psi_0\rangle = \gamma \sum_n \frac{(\alpha p^\dagger)^n}{n!} |0\rangle = \gamma \prod_i e^{\alpha\beta_i a_i^\dagger}|0\rangle \tag{2}$$

where α is a complex number sufficiently large that the pulse contains a large number of photons. The interaction Hamiltonian is given by

$$H' = \sum_i \sum_{jk} \epsilon_{ijk}(b_k^\dagger b_j^\dagger b_i + c_k^\dagger c_j^\dagger c_i) + h.c. \tag{3}$$

Here the operators b and c annihilate photons in the two paths through the interferometer and ϵ_{ijk} is a coefficient of no interest.

The intensity at the detector can then be shown to be

$$\langle I(x,t)\rangle = \langle E^- E^+\rangle = \gamma' \sum_{ij} \sum_{i'j'} e^{i[(k_{j'}-k_j)x-(\omega_{j'}-\omega_j)t]} \epsilon^*_{ijk} \epsilon_{i'j'k'} \beta_i^* \beta_{i'} \tag{4}$$
$$(\alpha^*\alpha\langle b_k|b_{k'}\rangle + \alpha'^*\alpha'\langle c_k|c_{k'}\rangle - \alpha'^*\alpha\langle c_k|b_{k'}\rangle - \alpha^*\alpha'\langle b_k|c_{k'}\rangle)$$

The last two terms are the only ones that depend on the relative phase, as reflected by the coefficients α and α', and are proportional to the inner product of two states containing a photon in two different paths, which is zero. Thus the entanglement of the original photon with a secondary photon in one path or the other is responsible for destroying the interference pattern, as expected.

It is interesting to note, however, that it is not possible, even in principle, to determine which path a photon has taken, since the quantum uncertainty in the energy and number of photons in the coherent state of eq. (2) makes it impossible to associate the detected photons with individual secondary photons.

Any classical description of this experiment must be based on a nonlinear model, since a linear system cannot produce any change in the frequency of the light. With this in mind, consider a simple model consisting of three nonlinearly-coupled harmonic oscillators:

$$\begin{aligned} \ddot{x}_i &= -\omega_i^2 x_i + 4\epsilon(x_i - x_k)^3 - \eta\dot{x}_i + d(t) \\ \ddot{x}_j &= -\omega_j^2 x_j - 4\epsilon(x_k - x_j)^3 - \eta\dot{x}_j \\ \ddot{x}_k &= -\omega_k^2 x_k - 4\epsilon(x_i - x_k)^3 + 4\epsilon(x_k - x_j)^3 - \eta\dot{x}_k \end{aligned} \tag{5}$$

The three frequencies ω_i, ω_j, and ω_k correspond to the frequencies of the three light beams in Figure 2, ϵ and η are adjustable constants, and d(t) represents an external driving field. This model is not intended to provide a realistic description of the response of an atom to an incident beam of light but does illustrate the kind of behavior that can occur in classical systems. It may be worth noting, for example, that the Coulomb force is a nonlinear function of the separation of two particles and naturally gives rise to nonlinear effects of this kind.

It was assumed that a nonlinear system of this kind is located in each path of Figure 2 and the above set of equations was solved numerically[3] for the case in which the incident field has frequency ω_d. The resulting power spectral density for a sufficiently intense incident field is shown in Figure 3 and has a sharp peak at the incident frequency as well as a somewhat broader peak corresponding to fluorescence, as in the quantum description. The phase-space trajectory of the out-going field x_j is plotted in Figure 4, where it can be seen that the motion is chaotic and unpredictable. This randomizes the phase of the field in a manner that is also similar to the quantum treatment.

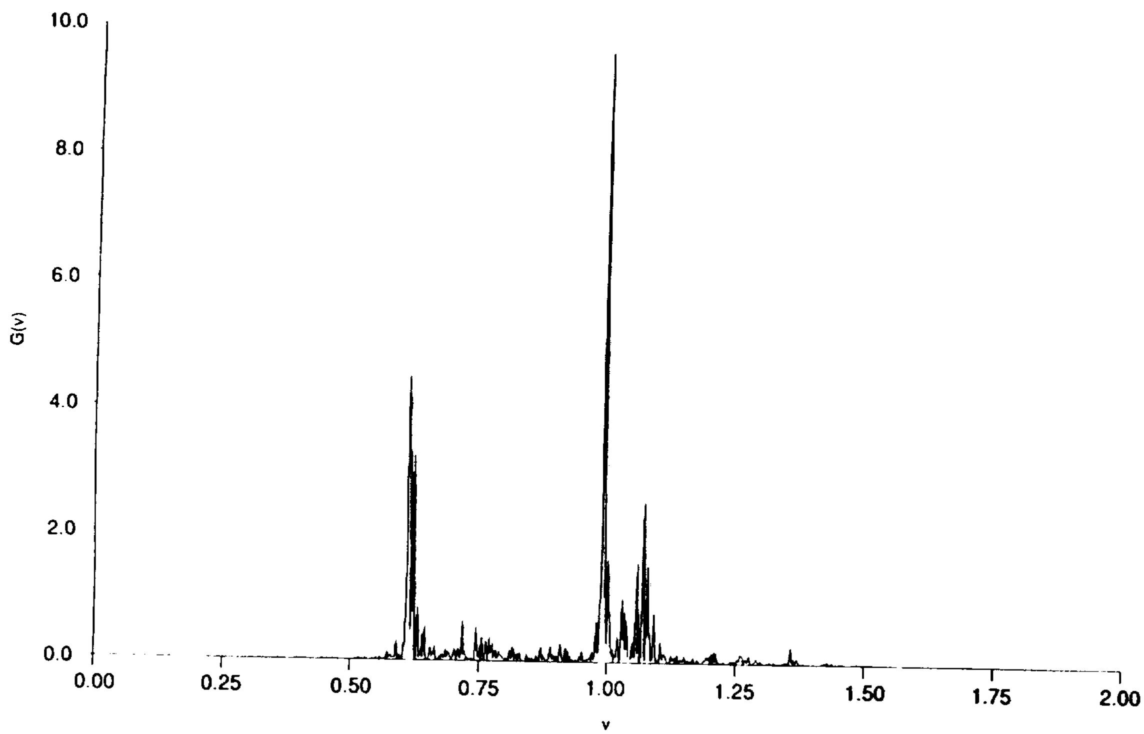

Fig. 3. Power spectral density $G(\nu)$ obtained from the classical model, showing the classical analog of fluorescence.

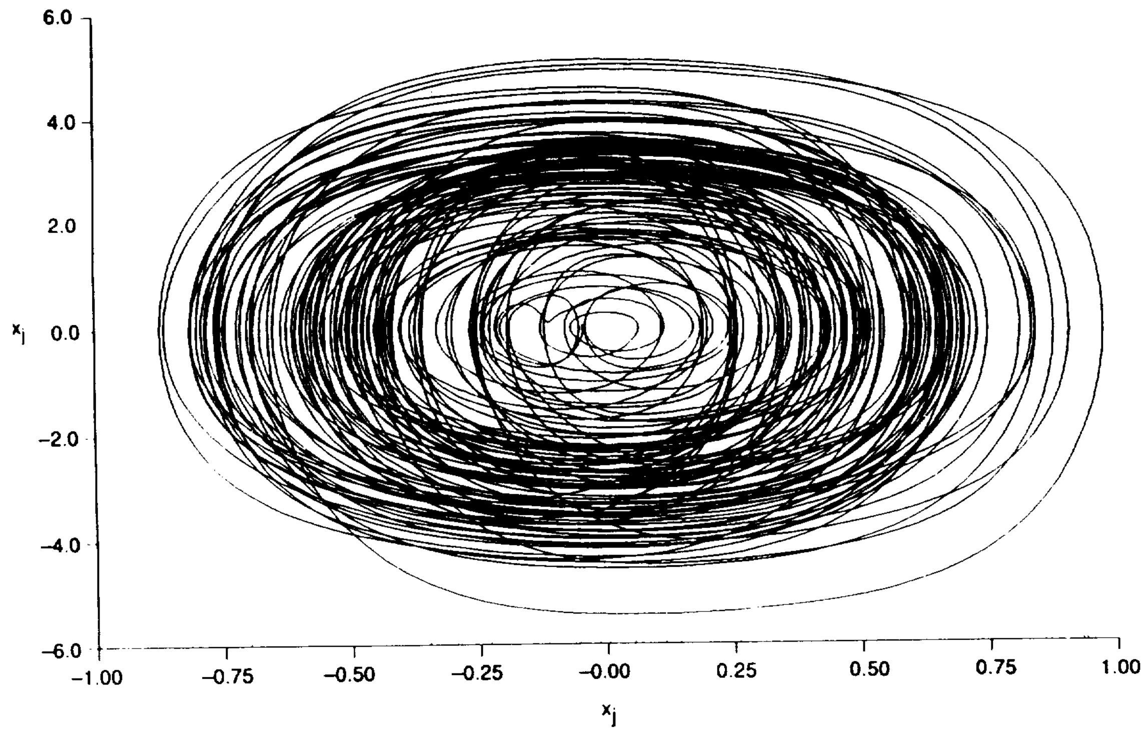

Fig. 4. Chaotic phase-space trajectory from the classical model, which randomizes the phase and destroys the interference pattern.

In the limit of low drive intensities the classical model produces a coherent response with no fluorescence. At sufficiently high intensities, chaos produces fluorescence at two frequencies that are analogous to the secondary photon and forward-propagating photons in Figure 2, both of which have random phase. Intermediate intensities produce more complicated behavior, including partial coherence at rational fractions of the drive frequency.

Thus the classical model gives loss of the interference pattern due to chaos in the macroscopic limit of high intensities. This suggests that there is at least a loose connection between quantum noise and classical chaos. It is important to note, however, that the classical model produces a random phase only for sufficiently high intensities, whereas a proper quantum-mechanical treatment eliminates the coherence for arbitrarily low intensities.

In many systems of this kind it is possible to implement a "quantum eraser" to restore the interference pattern[1]. This can be accomplished by letting the entangled secondary systems propagate in time, measuring their state at some subsequent time, and selecting only those events for which the secondary systems were found to be in the same final state. For example, a quantum eraser can be implemented for the micro-maser cavity experiment shown in Figure 1 by connecting the two cavities with a small hole containing an atom and then selecting only those events for which the photon in one cavity or the other was absorbed by this atom.

Surprisingly enough, it may be possible to perform a similar procedure in the classical model discussed above. Suppose we consider a subset of the phase-space trajectories for which the other (non-detected) variables are the same in the two paths, i.e.

$$x_1 = x_1' \qquad (6)$$
$$x_2 = x_2'$$

where the primed and unprimed variables refer to the two different paths. In that case, it seems likely that

$$x_3 = x_3' \qquad (7)$$

If so, the out-going fields would be the same in the two paths and the coherence would be restored.

3. Spontaneous Emission Noise

The random phase associated with spontaneous emission of a photon by an atom is often attributed to vacuum fluctuations. Once again, this may seem to be inherently quantum-mechanical in nature. But returning to the example shown in Figure 2, it can be seen that both photons emitted by an atom in one path or the other of that interferometer are emitted by spontaneous emission. The classical model discussed above gave a random phase for both of these fields due to classical chaos in the limit of high field intensities, which is in qualitative agreement with the quantum-mechanical result.

This further suggests that there may be some connection between quantum noise and classical chaos. It must be kept in mind, however, that the classical model cannot produce these kinds of results in the limit of low intensities.

4. Nonlocal Effects

The preceding discussion suggests that certain kinds of quantum phase noise may have a classical analogy in the form of chaos. This analogy can only be taken so far, however, since the models used do not provide a realistic description of an atom and are qualitatively similar to the quantum-mechanical treatment only in the limit of high intensities.

In addition, quantum systems can exhibit nonlocal effects that violate Bell's inequality and obviously have no classical analog. Such effects are not limited to low-intensity fields, as can be illustrated by considering the generalization of two-photon interferometry[4] to high-intensity fields as illustrated in Figure 5. A somewhat similar situation involving photon polarizations has also been discussed by Reid and Munro[5].

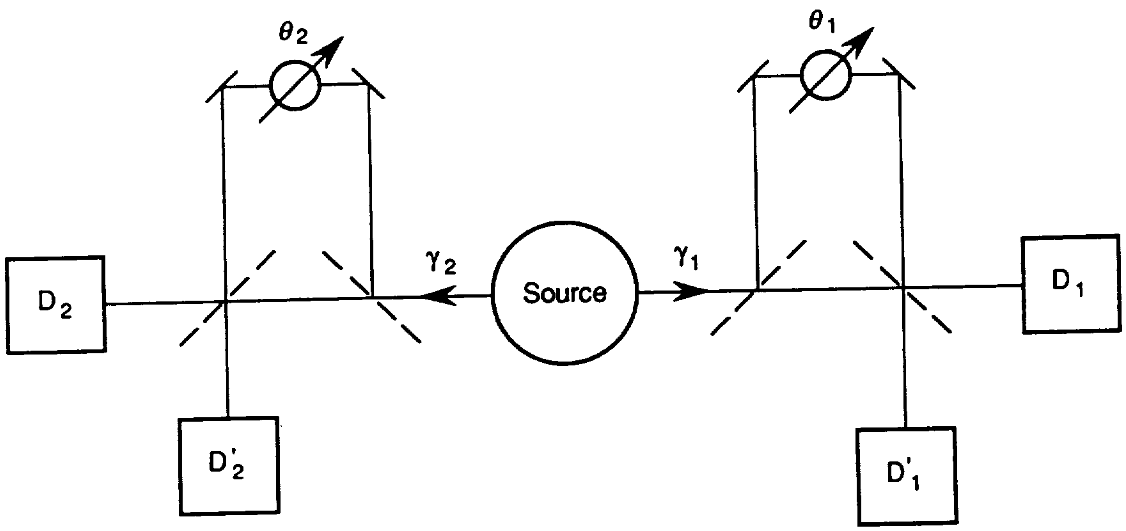

Fig. 5. Nonlocal interferometer consisting of two identical interferometers with a short path and a long path, capable of operation with high-intensity fields.

Nonlocal interferometry with high-intensity fields has been discussed in detail elsewhere[6] and only the main results will be reviewed here. Consider a quantum state of the electromagnetic field given by

$$|\Psi\rangle = \gamma \sum_n \frac{(\alpha c^\dagger)^n}{n!} |0\rangle = \gamma e^{\alpha c^\dagger} |0\rangle \tag{8}$$

where

$$c^\dagger = \sum_k f_k a_k^\dagger b_{k_0-k}^\dagger \tag{9}$$

Here $c^\dagger$ creates a pair of entangled photons in two paths via photon creation operators $a^\dagger$ and $b^\dagger$, γ is a normalization constant, α is a large complex number, and the coefficients f_k describe the effects of filters inserted into the two beams.

Although eq. (8) resembles a coherent state, its properties are quite different. The probability P_1 of detecting a pair of coincident photons in the corresponding output ports of the two interferometers of Figure 5 can be shown to be given by

$$P_1 = \eta \cos^2\left[\frac{\phi_A+\phi_B+\omega_0\Delta t}{2}\right] \tag{10}$$

This is the same result obtained previously for the weak-field case but here the field can be extremely intense and contain a large number of photons.

The probability P_N of detecting N pairs of coincident photons in the corresponding output ports of the two interferometers is

$$P_N = N!\ \eta^N \cos^{2N}\left[\frac{\phi_A+\phi_B+\omega_0\Delta t}{2}\right] = N!\ P_1^N \tag{11}$$

Eq. (11) also violates Bell's inequality. The factor of N! is due to the different ways in which photons can pair with each other and greatly enhances the probability of detecting a large number of pairs. No single pnoton detectors are required to observe such events, which correspond to large bursts of energy in the corresponding interferometer ports and which could be observed, at least in principle, with a bolometer. These effects are truly macroscopic in nature in that sense.

It is also possible to consider an EPR paradox involving quantum phase measurements performed on high-intensity fields with initially uncertain phases. Both classical and non-classical effects are obtained, as described elsewhere[7].

5. Summary

Several sources of quantum phase uncertainty have been considered in the limit of high field intensities where a comparison with a classical treatment is possible. It was found that classical analogies exist for the loss of coherence due to which-path information as well as the quantum noise associated with spontaneous emission. In both of these cases classical chaos randomizes the phase in a manner that is at least qualitatively the same as in the quantum description.

This suggests that there may be a loose connection between quantum noise and classical chaos. The classical treatment is only valid in the limit of high intensities, however, which is not too surprising in that classical physics would not be expected to provide an adequate description at the quantum level. In addition, violations of Bell's inequalities can also occur for high-intensity fields. Nevertheless, there does appear to be an analogy between quantum noise and classical chaos.

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